On expressibility and axiomatization of modal logics of distances

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Joint work with Gabriel Agnew, Uzias Gutierrez-Hougardy, John Harding, Hannah Himelright, Jackson West, and Andrew Meléndrez Zerwekh on the project "Research Training Group in Logic and Its Application" 2023 – 2024.

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Modal language is known to be an efficient formalism for working with relational structures: various properties of relations can be captured by this language, and at the same time, the resulting theories are often decidable. In particular, it is of interest to consider modal logics of relations induced by distance in a metric space. This direction has been studied since 2000s, see, e.g., [1, 2, 3, 4, 5, 6].

In this talk I will announce results, recently obtained by our group in this direction. We had two branches in the project. The first was devoted to theoretical problems. In a metric space, two points are defined to be *close*, if they are at a distance less than 1 (or any other fixed positive number), and *far*, if they are at a distance greater than 1. The corresponding modal logics are called *logics of closeness* and *logics of farness*. For logics of closeness and farness considered on various metric spaces, new results on expressibility, complete axiomatization, finite axiomatizability, and decidability were obtained.

Another branch of the project focused on software for modal logics of finite relational structures. In particular, the following decision problem was considered: given a natural number m and two finite structures, to decide whether the m-variable fragments of their logics coincide (this problem is related to the property of finite axiomatizability). An algorithm solving the m-equivalence problem was developed.

This work was supported by NSF Grant DMS - 2231414.

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